



THE UNIVERSITY OF TEXAS AT DALLAS

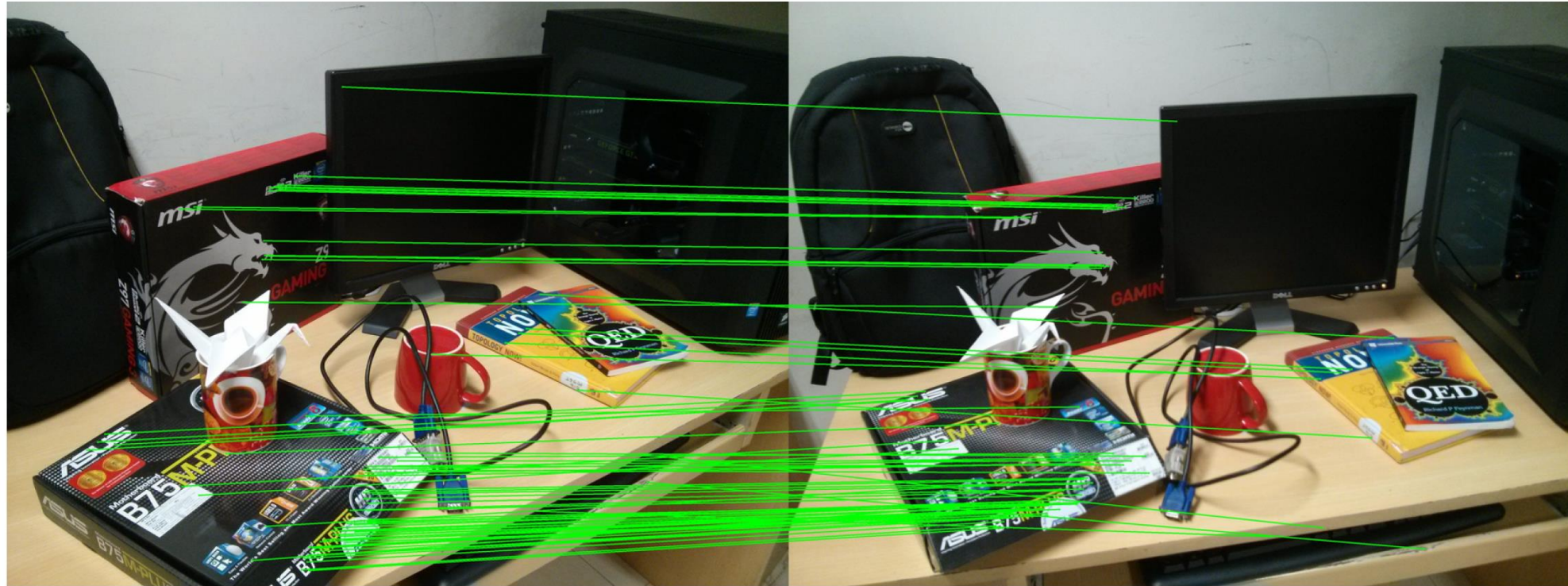
Feature Detection and Matching: Detectors and Descriptors II

CS 6384 Computer Vision

Professor Yapeng Tian

Department of Computer Science

Feature Detection and Matching

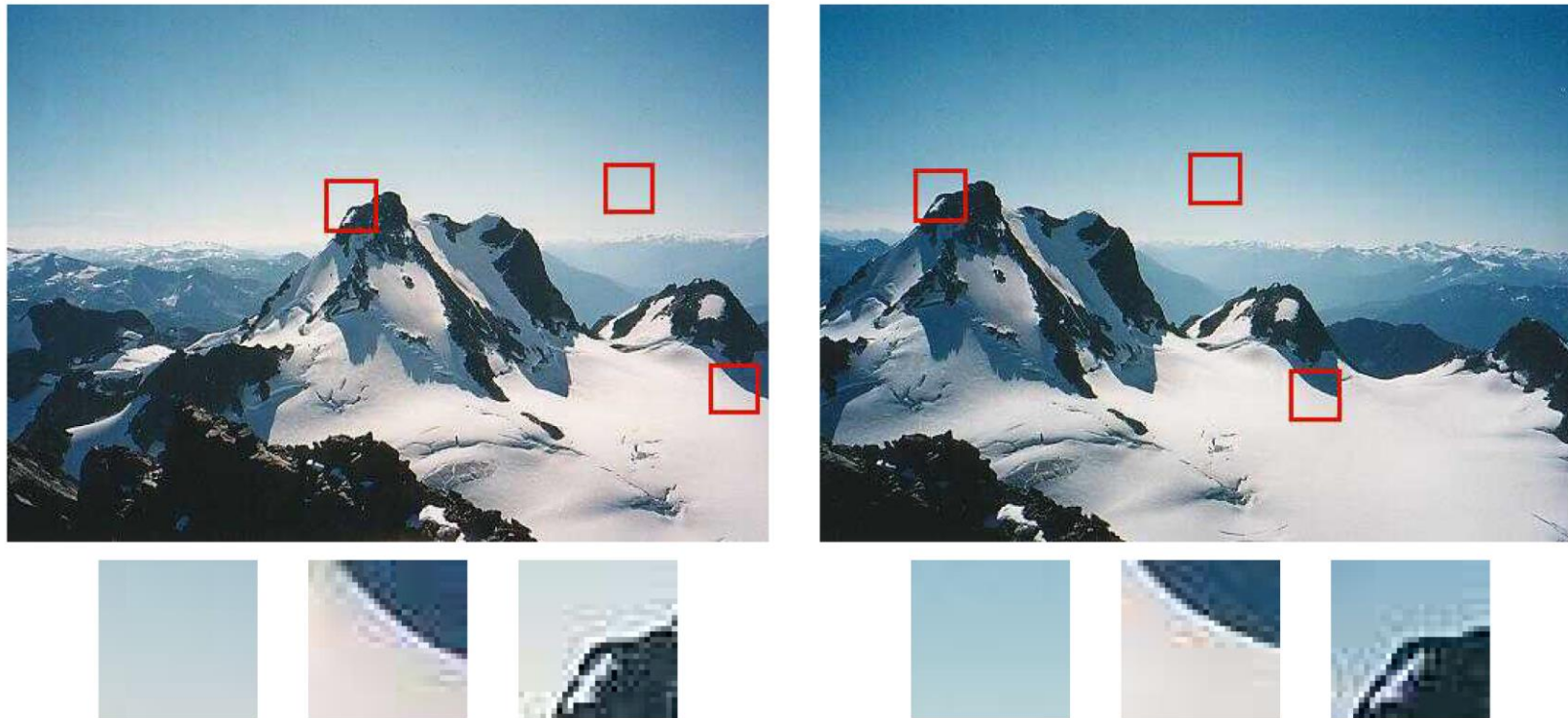


Geometry-aware Feature Matching for Structure from Motion Applications. Shah et al, WACV'15

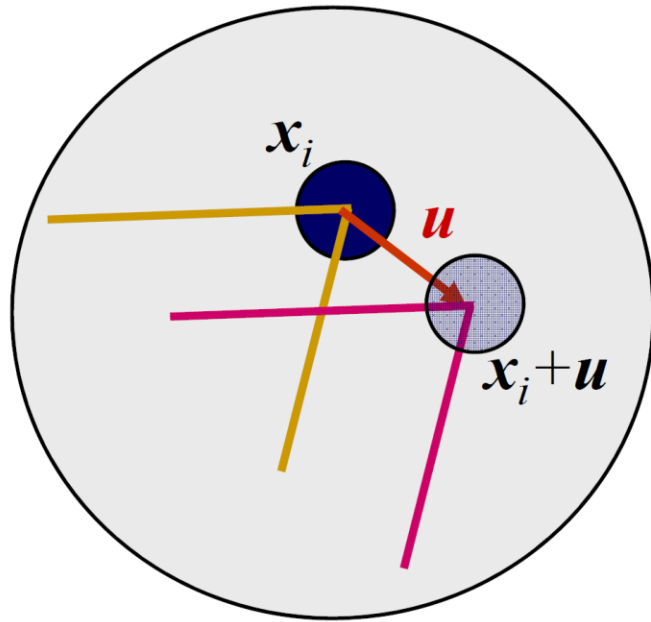
Applications: stereo matching, image stitching, 3D reconstruction, camera pose estimation, object recognition

Feature Detectors

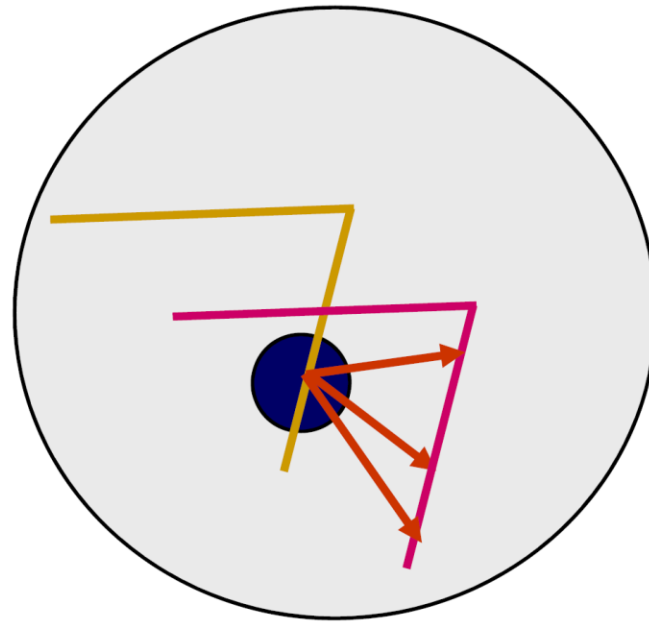
How to find image locations that can be reliably matched with images?



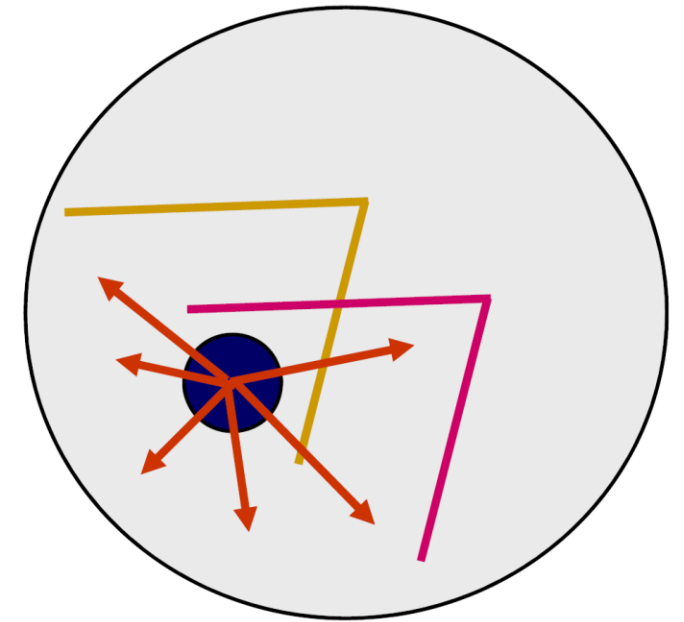
Feature Detectors



(a)
Corner



(b)
Edge



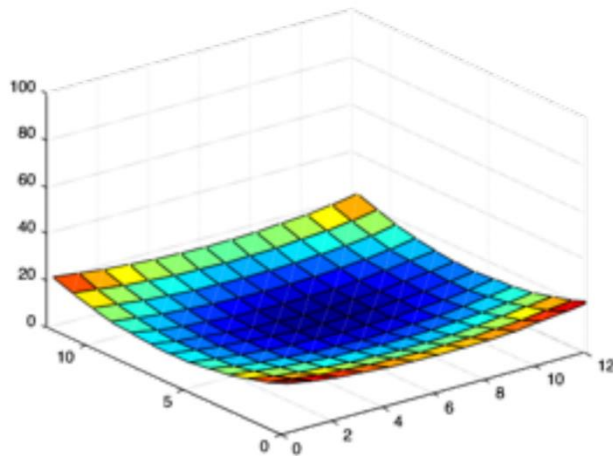
(c)
Textureless region

Harris Corner Detector

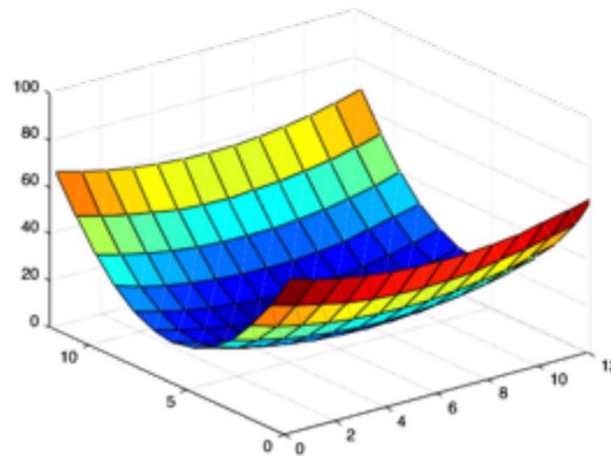
$$f(\Delta x, \Delta y) \approx \sum_{x,y} w(x,y) (I_x(x,y)\Delta x + I_y(x,y)\Delta y)^2$$

$$f(\Delta x, \Delta y) \approx (\Delta x \quad \Delta y) M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

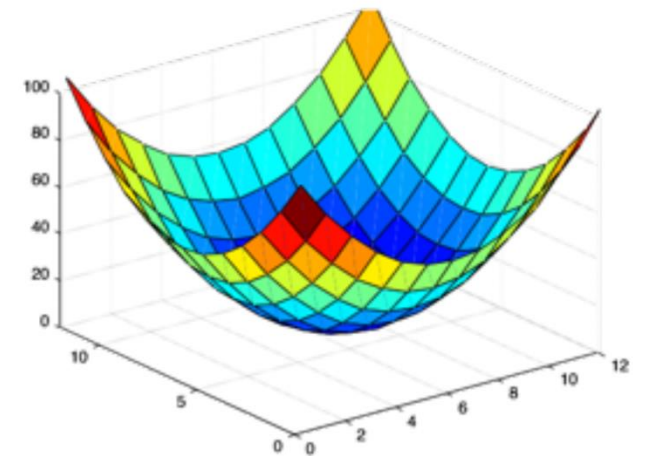
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$



Flat



Edge



Corner

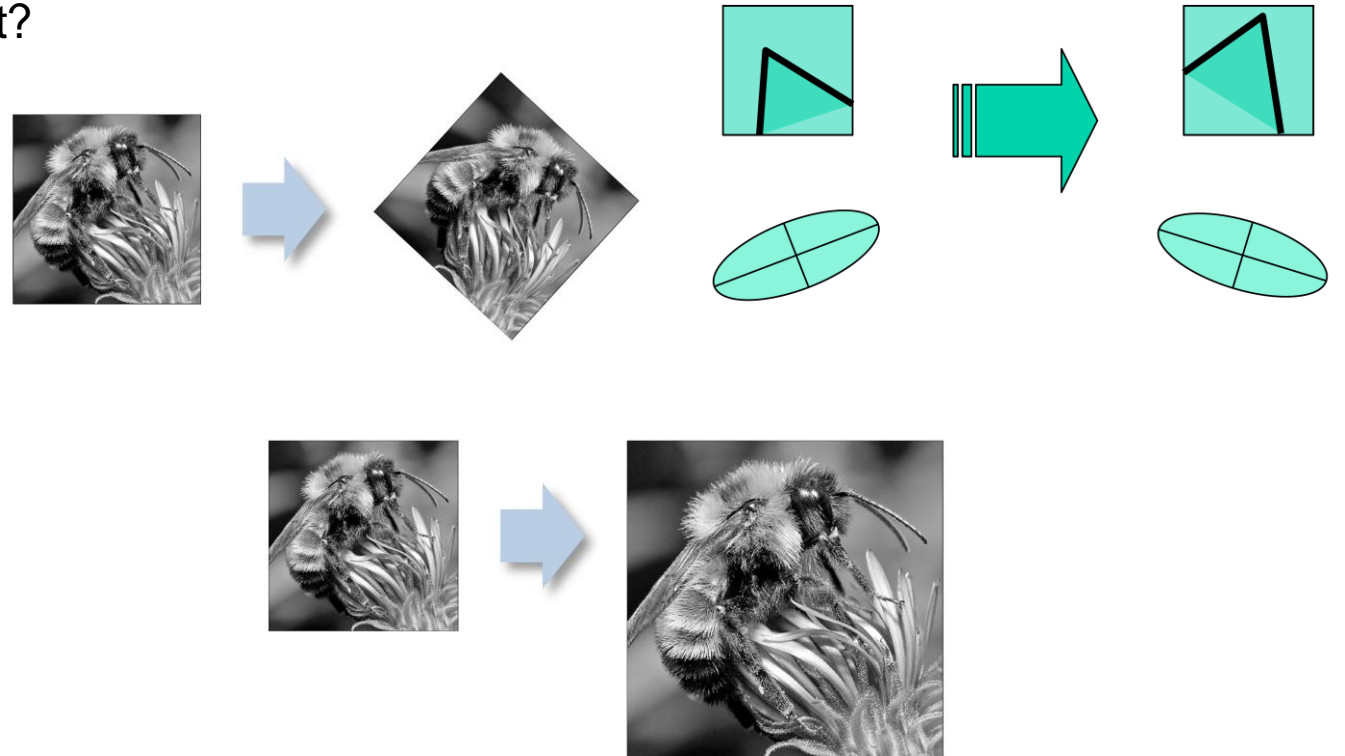
Invariance

Can the same feature point be detected after some transformation?

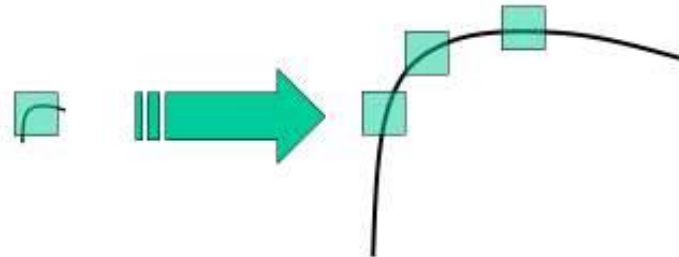
- Translation invariance
Are Harris corners translation invariant?

- 2D rotation invariance
Are Harris corners rotation invariant?

- Scale invariance
Are Harris corners scale invariant?

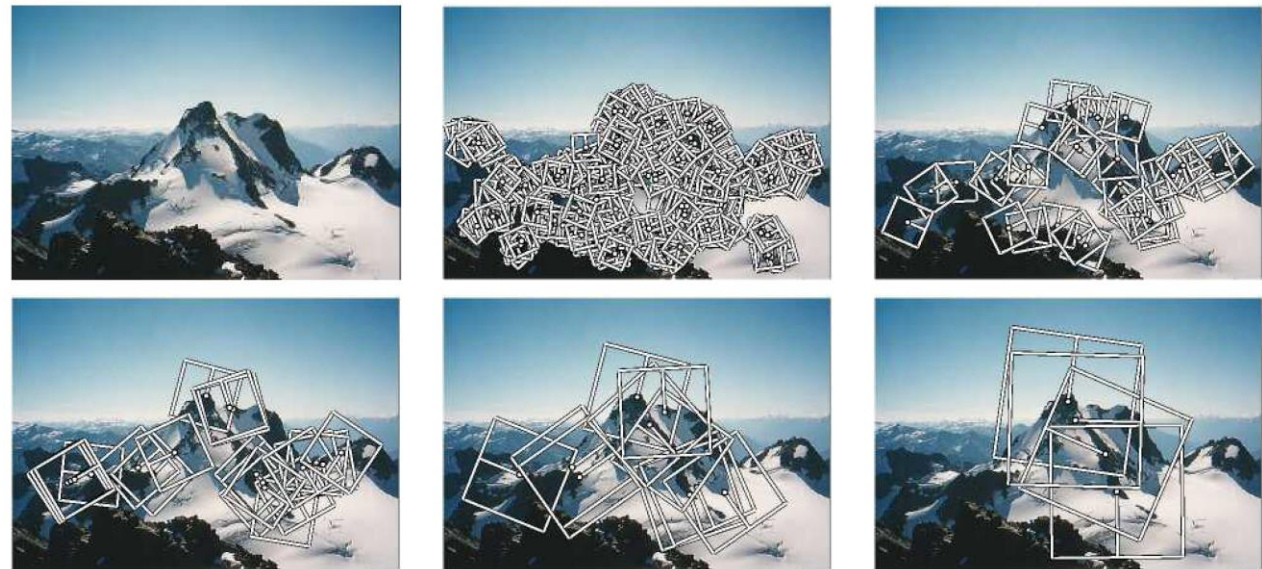
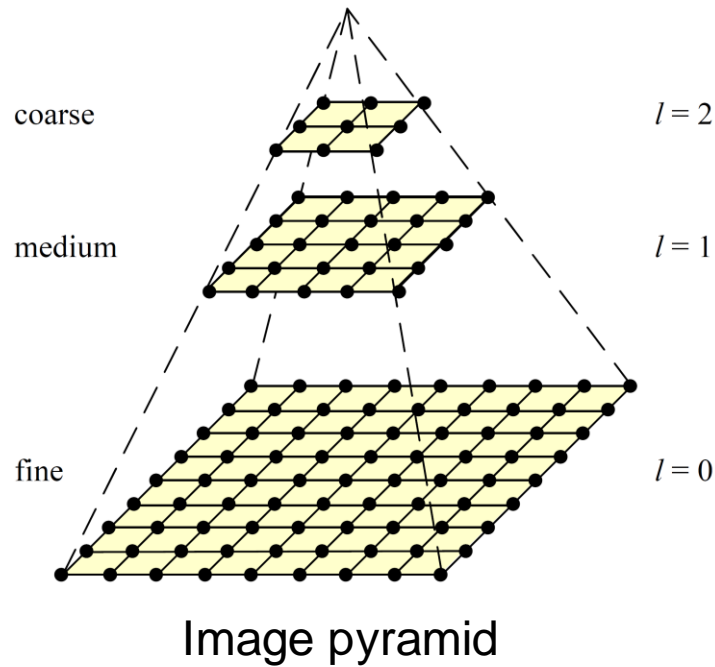


No



Scale Invariance

Solution 1: detection features in all scales, matching features in corresponding scale (for small scale change)

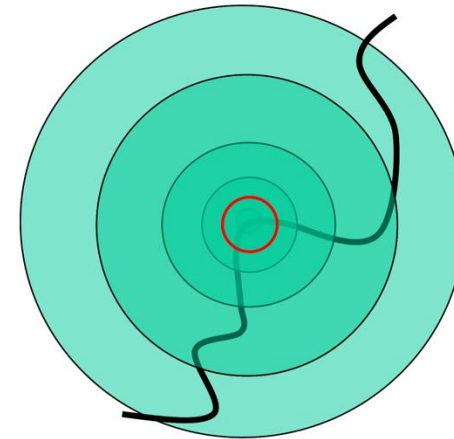
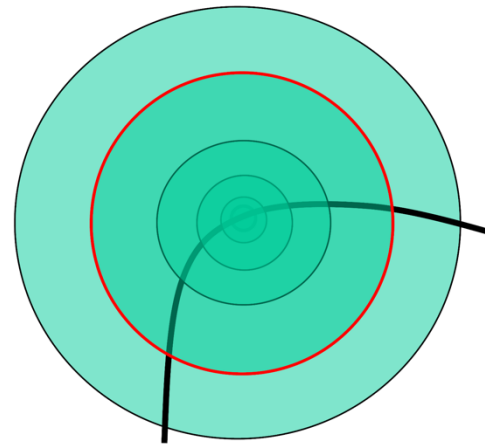


Multi-scale oriented patches (MOPS) extracted at five pyramid levels (Brown, Szeliski, and Winder 2005)

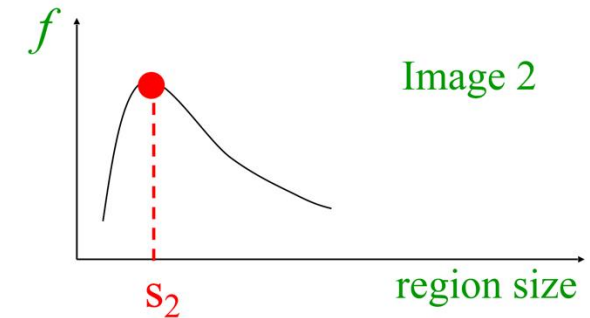
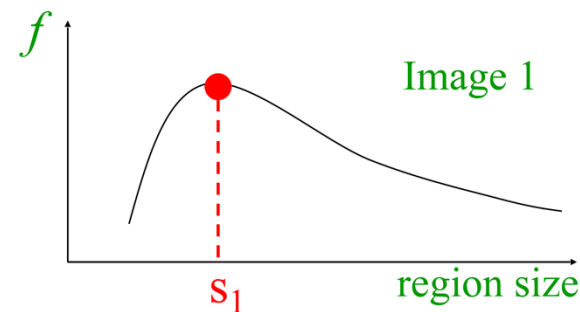
Scale Invariance

Solution 2: detect features that are stable in both location and scale

Intuition: Find local maxima in both position and scale



What filter can we use for scale selection?



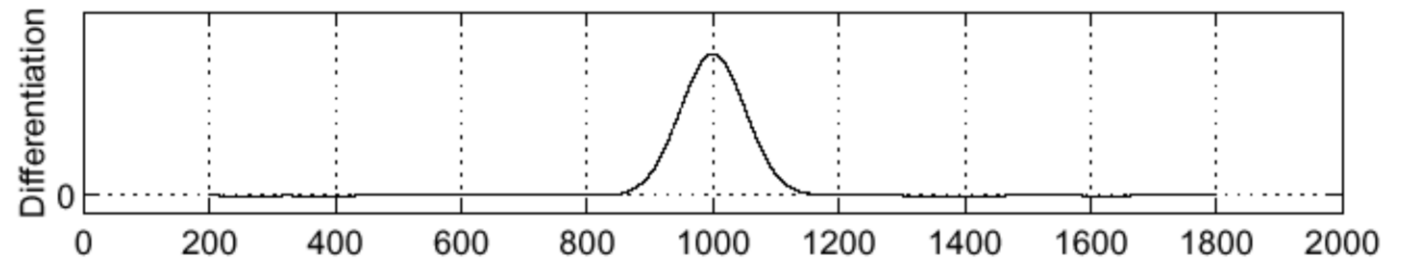
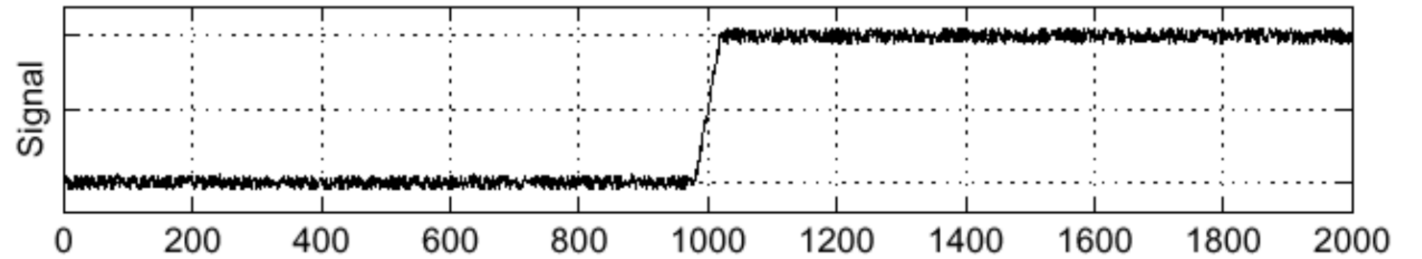
Recall Derivative Filter

Central difference

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

-1	0	1
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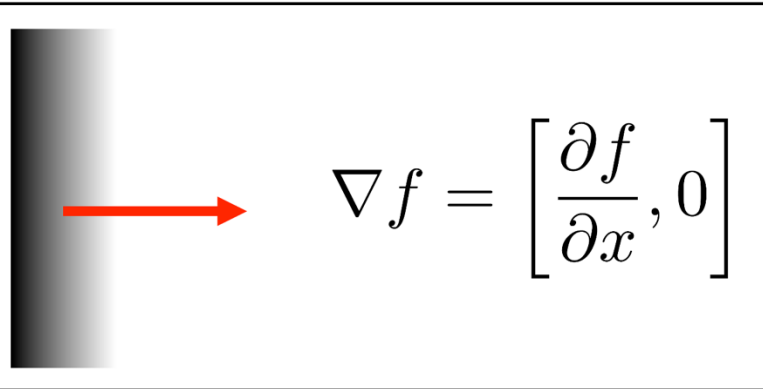
X derivative



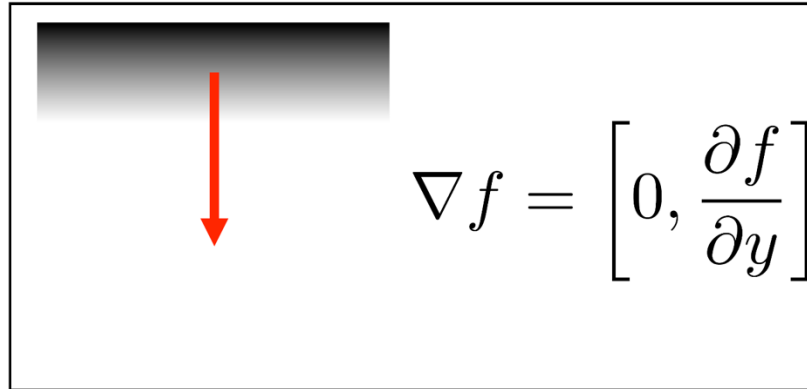
Find edge

Image Gradient

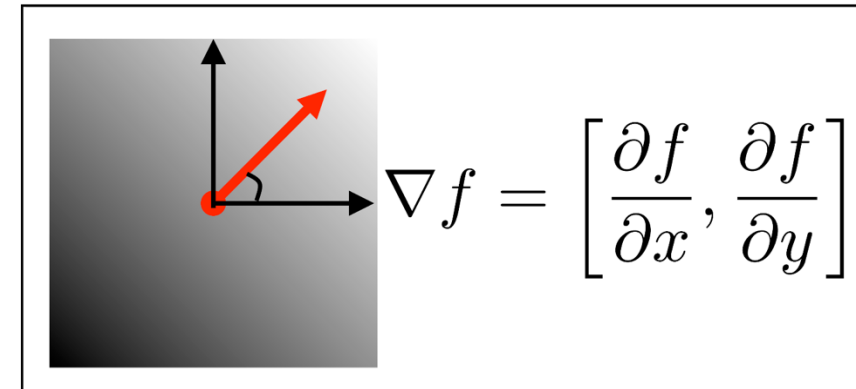
Gradient in x only



Gradient in y only



Gradient in both x and y



Gradient direction

$$\theta = \tan^{-1} \left(\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right)$$

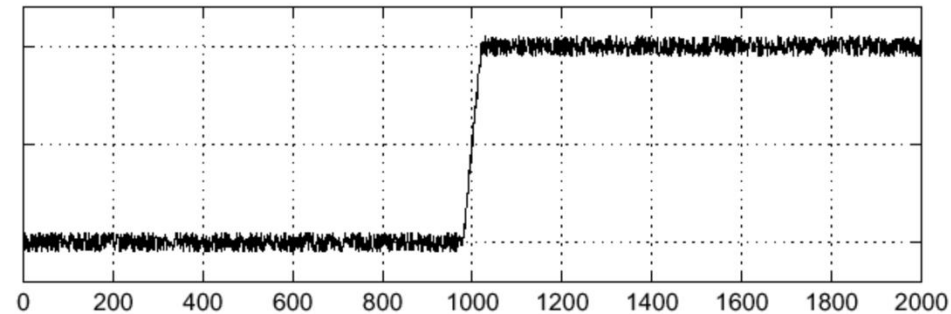
Gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

Signal Noises

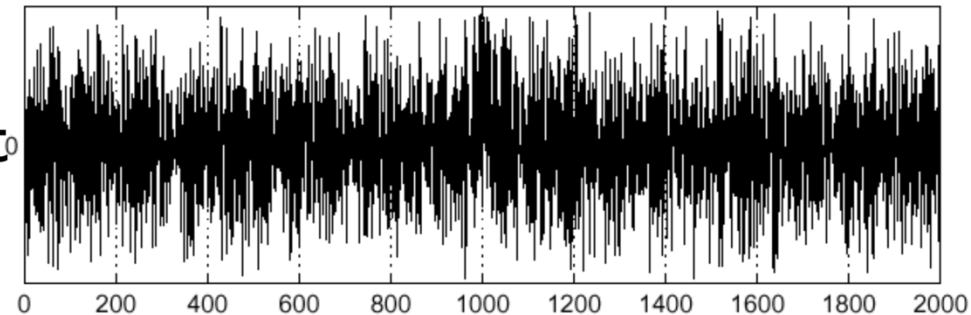
Derivative filters are sensitive to noises

Intensity plot



How to deal with noises?

Derivative plot



Gaussian Filter

Smoothing

$$1D \quad g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

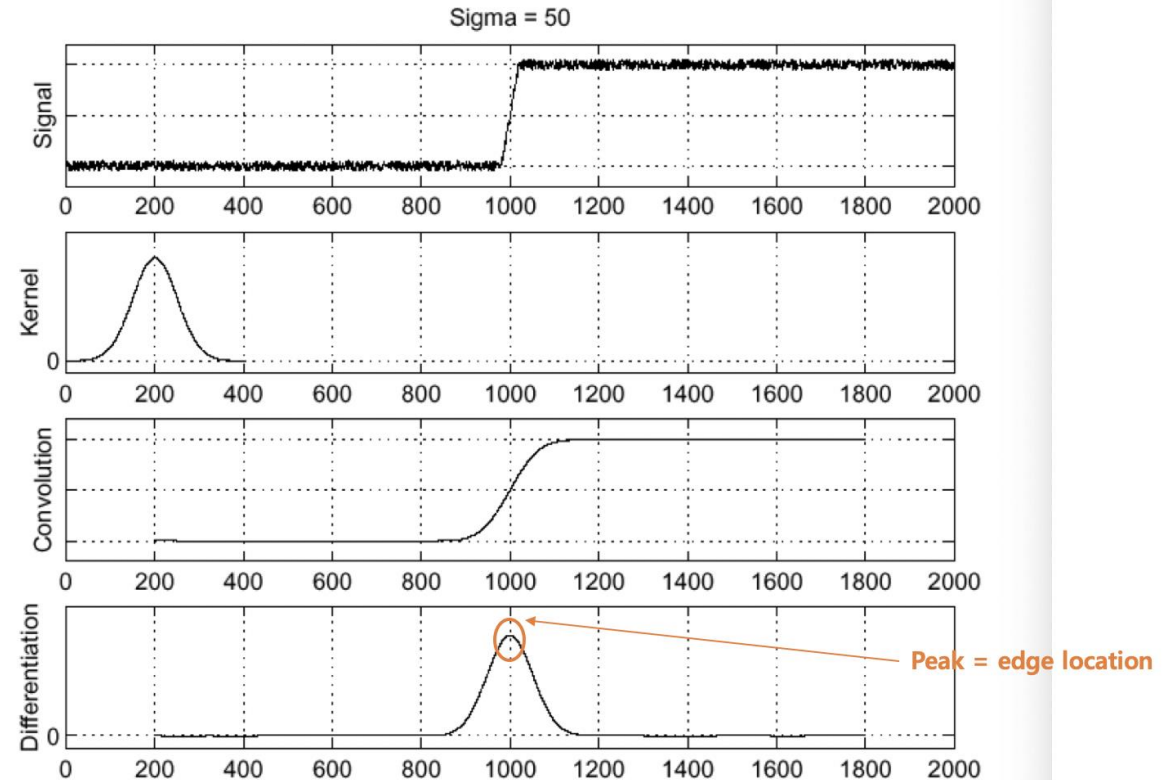
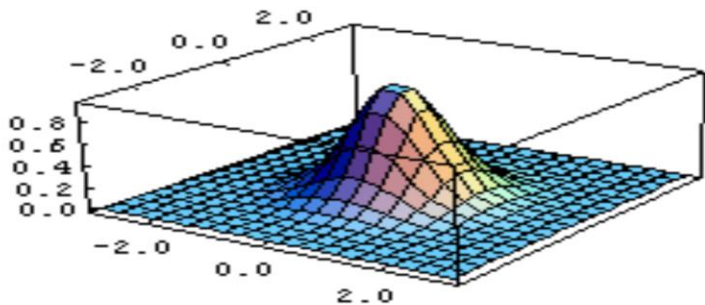
Image f

$$2D \quad g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Gaussian Filter h

Convolution $h \star f$

Derivative $\frac{\partial}{\partial x}(h \star f)$

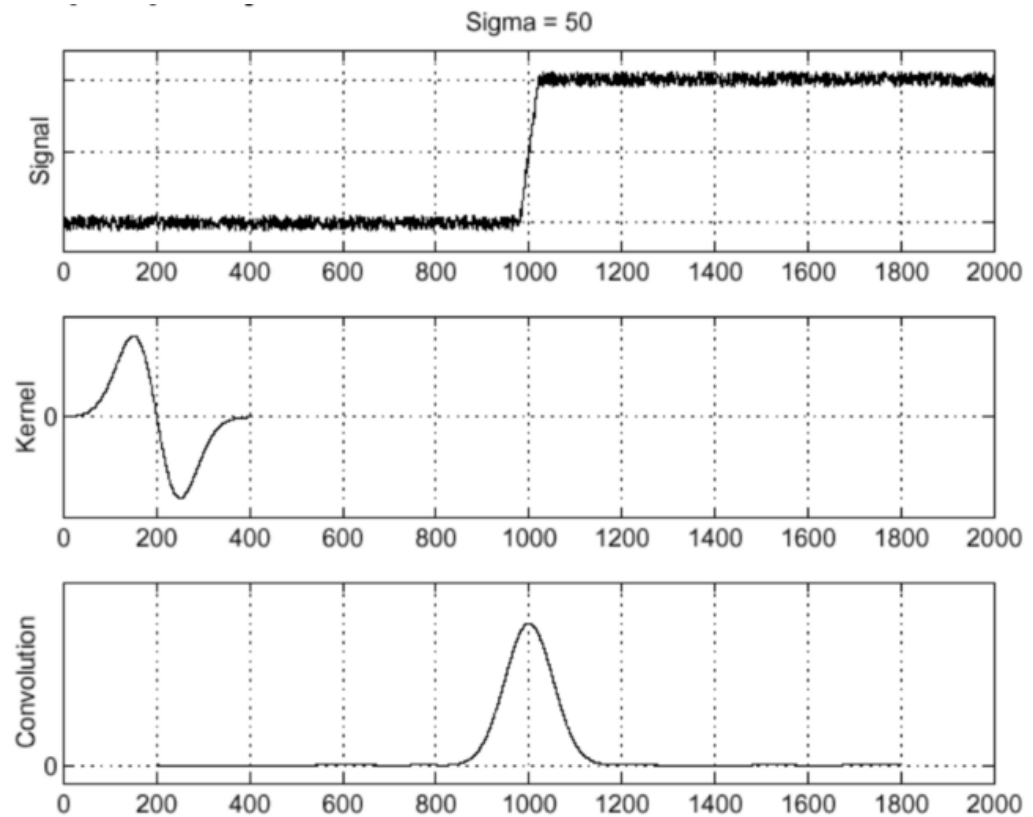


Derivative of Gaussian Filter

- Convolution is associative $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

Smoothing and derivative

$$(\frac{\partial}{\partial x}h) \star f$$



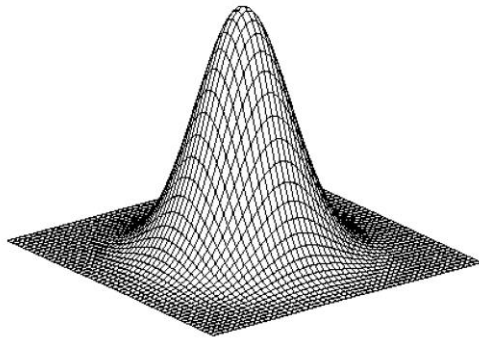
Derivative of Gaussian Filter

Convolution is associative

$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

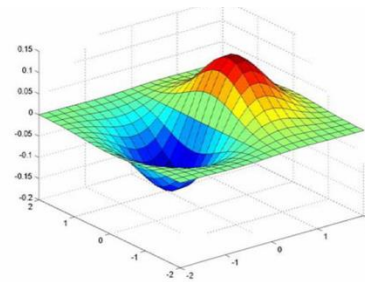
$$g_x(x,y) = \frac{\partial g(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$g_y(x,y) = \frac{\partial g(x,y)}{\partial y} = \frac{-y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

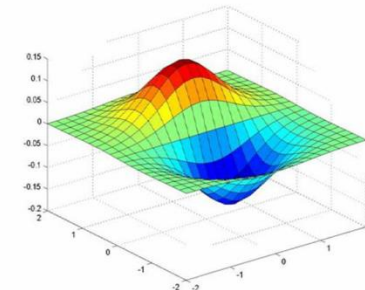


Gaussian

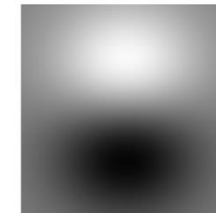
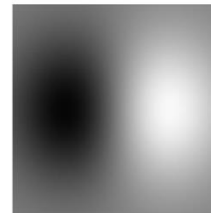
$$g(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



x-direction



y-direction



Laplace Filter

first-order
finite difference

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

Derivative filter

-1	0	1
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second-order
finite difference

$$f''(x) \approx \frac{\delta_h^2[f](x)}{h^2} = \frac{\frac{f(x+h)-f(x)}{h} - \frac{f(x)-f(x-h)}{h}}{h} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Laplace filter

1	-2	1
---	----	---

Laplace Filter

$$2D \quad \nabla^2 \mathbf{I} = \frac{\partial^2 \mathbf{I}}{\partial x^2} + \frac{\partial^2 \mathbf{I}}{\partial y^2}$$

1	-2	1
---	----	---

1D Laplace filter

0	1	0
1	-4	1
0	1	0

2D Laplace filter

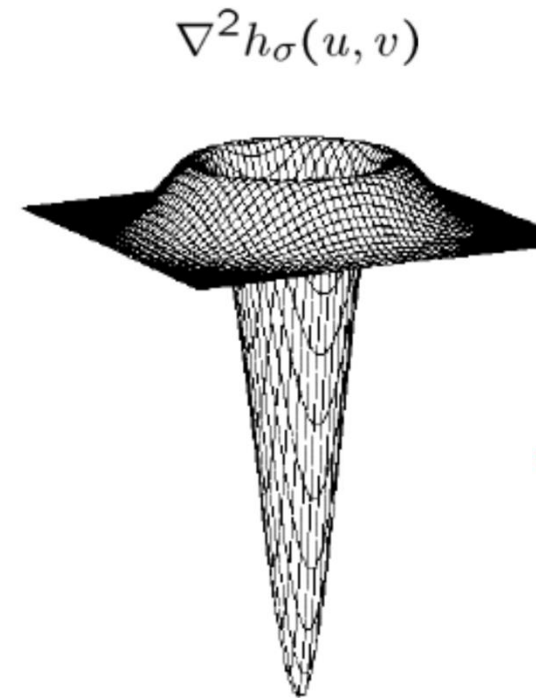
Laplacian of Gaussian Filter

$$\nabla^2 \mathbf{I} = \frac{\partial^2 \mathbf{I}}{\partial x^2} + \frac{\partial^2 \mathbf{I}}{\partial y^2}$$

$$\nabla^2 \mathbf{I} \circ g = \nabla^2 g \circ \mathbf{I}$$

$$\nabla^2 g = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} g(x, y)$$

Smoothing and second derivative

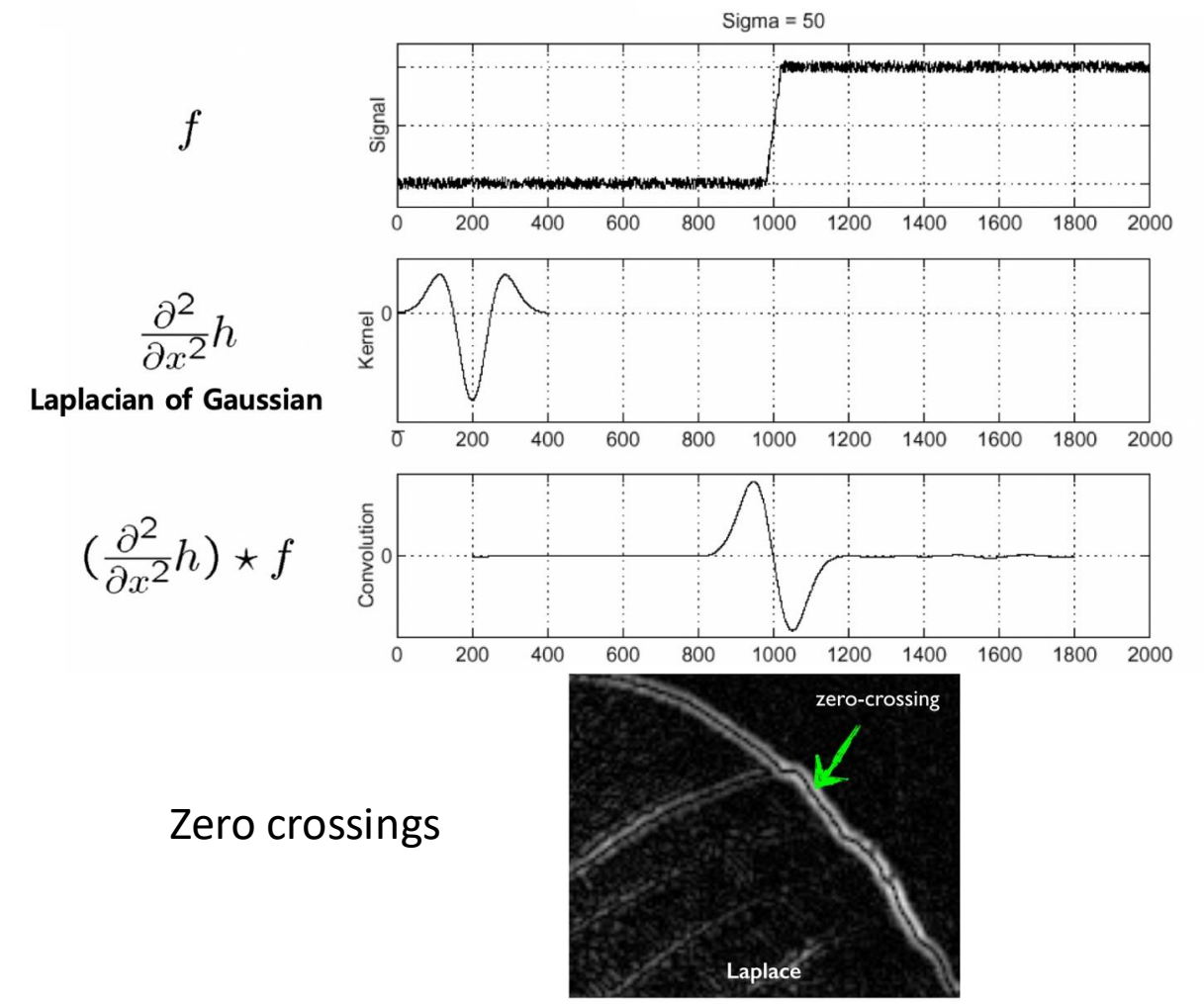
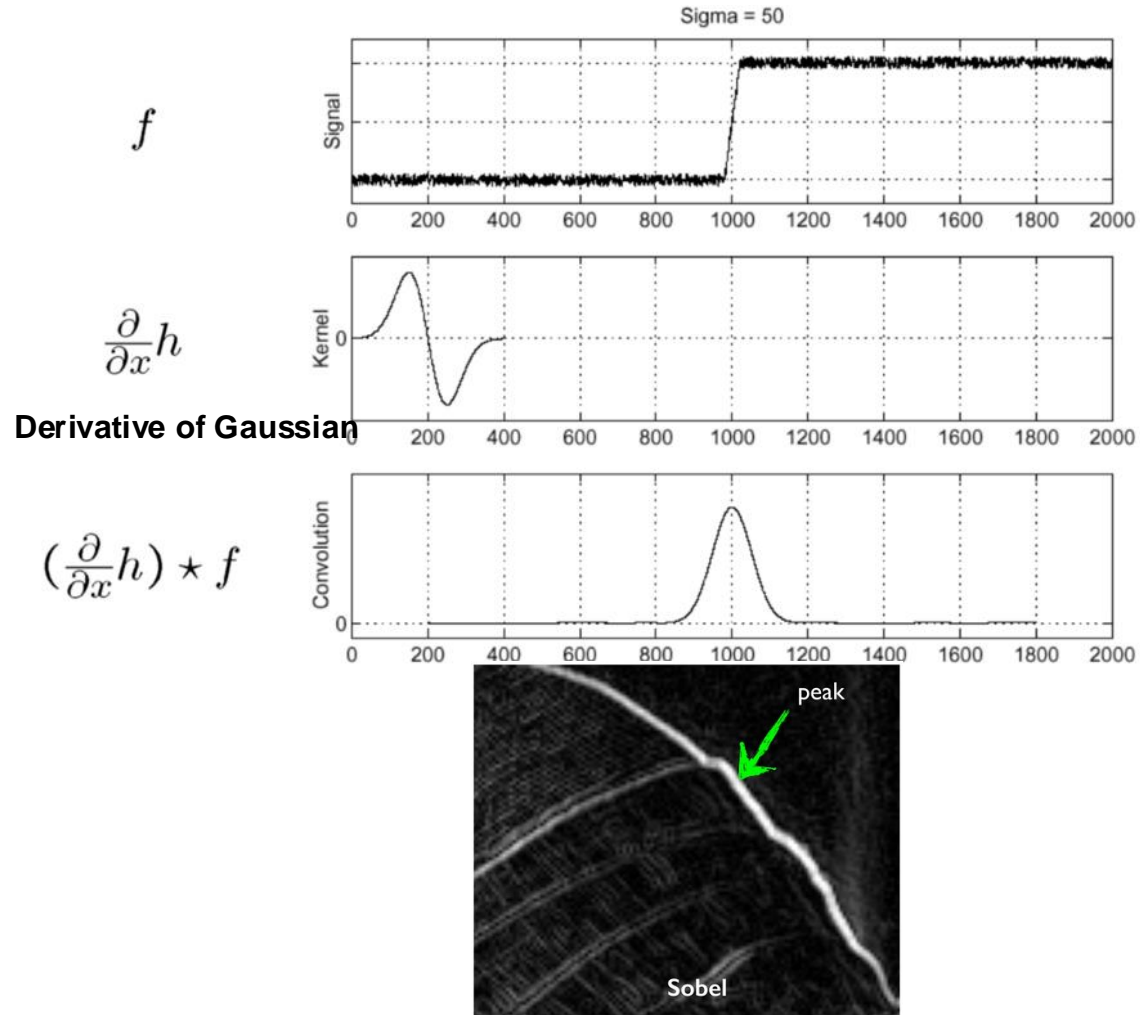


Laplacian of Gaussian



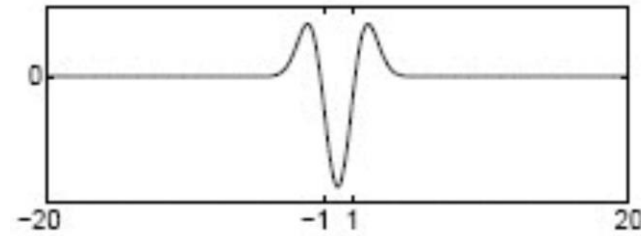
Mexican Hat Function

Laplacian of Gaussian Filter

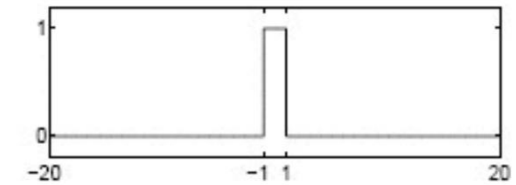
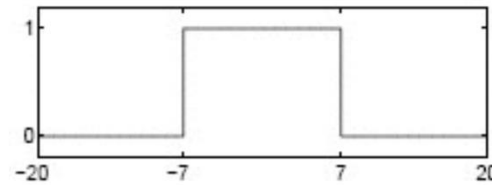
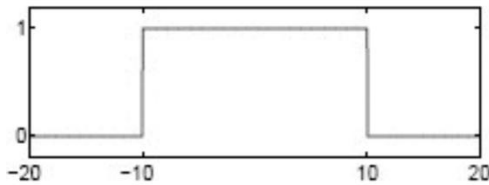


Laplacian of Gaussian for Scale Selection

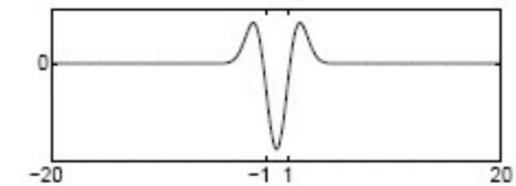
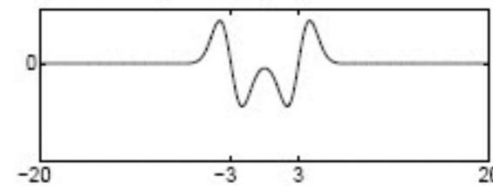
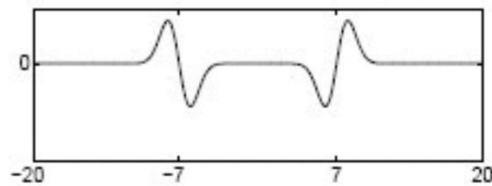
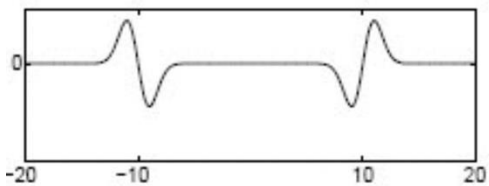
Laplacian filter



Original signal

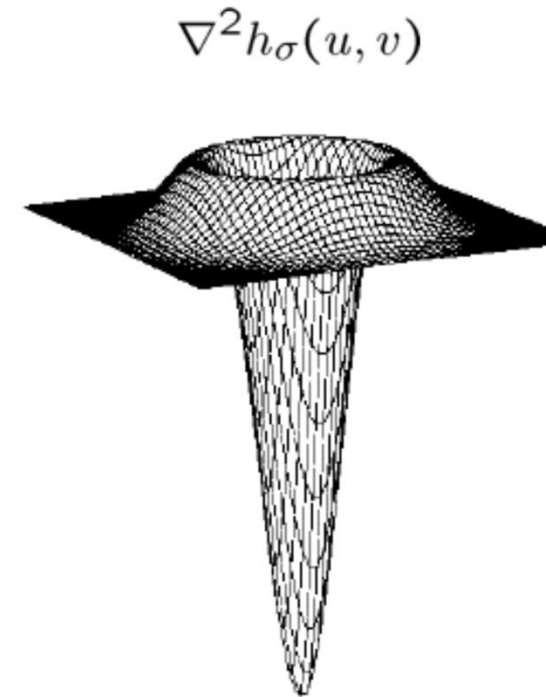
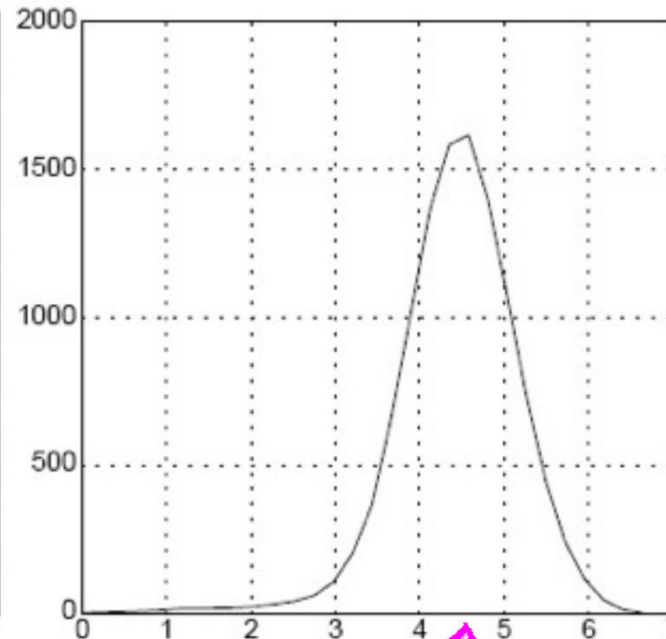
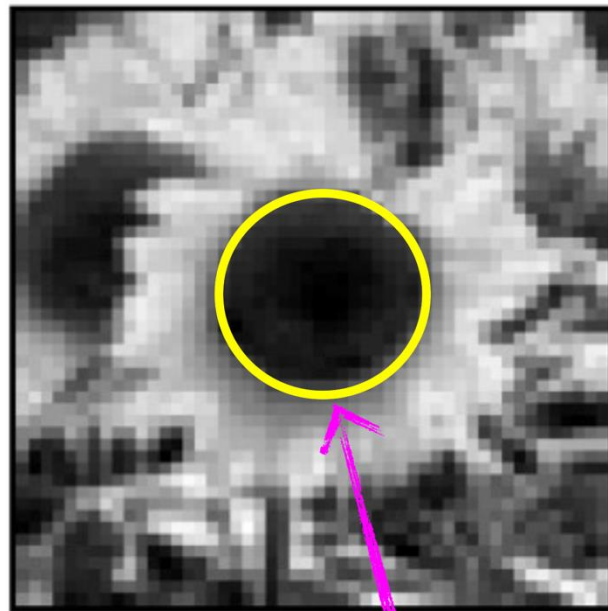


Convolved with Laplacian ($\sigma = 1$)



Highest response when the signal has the same **characteristic scale** as the filter

Laplacian of Gaussian for Scale Selection

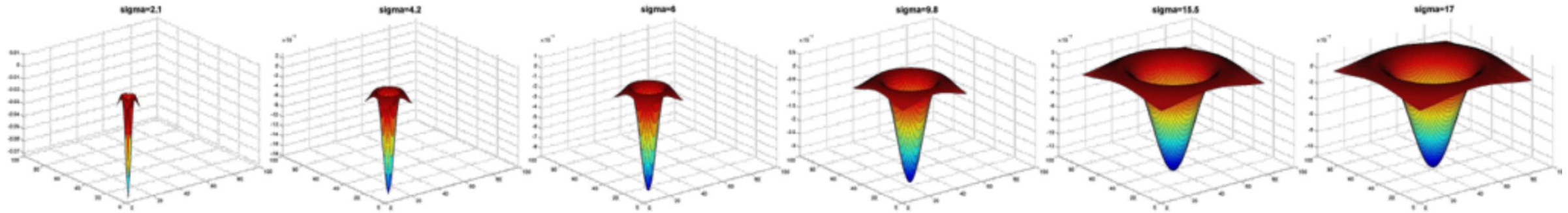


Laplacian of Gaussian

characteristic scale

Search over different scales σ

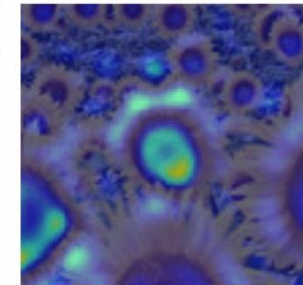
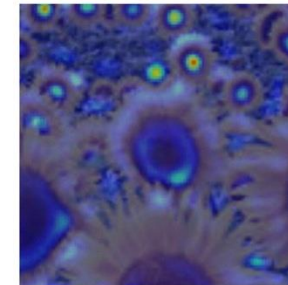
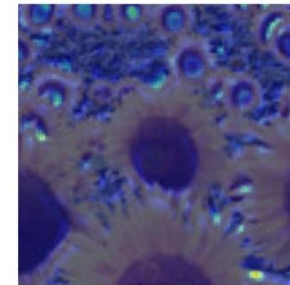
Laplacian of Gaussian for Scale Selection



2.1

4.2

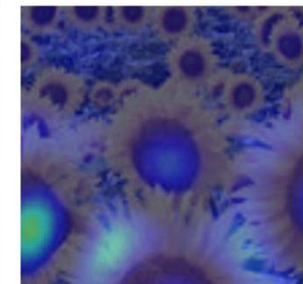
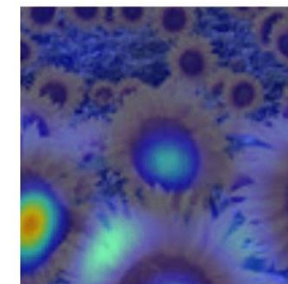
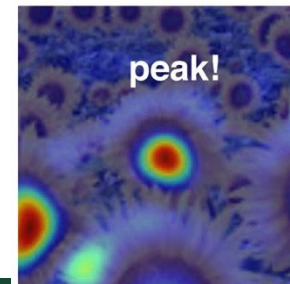
6.0



9.8

15.5

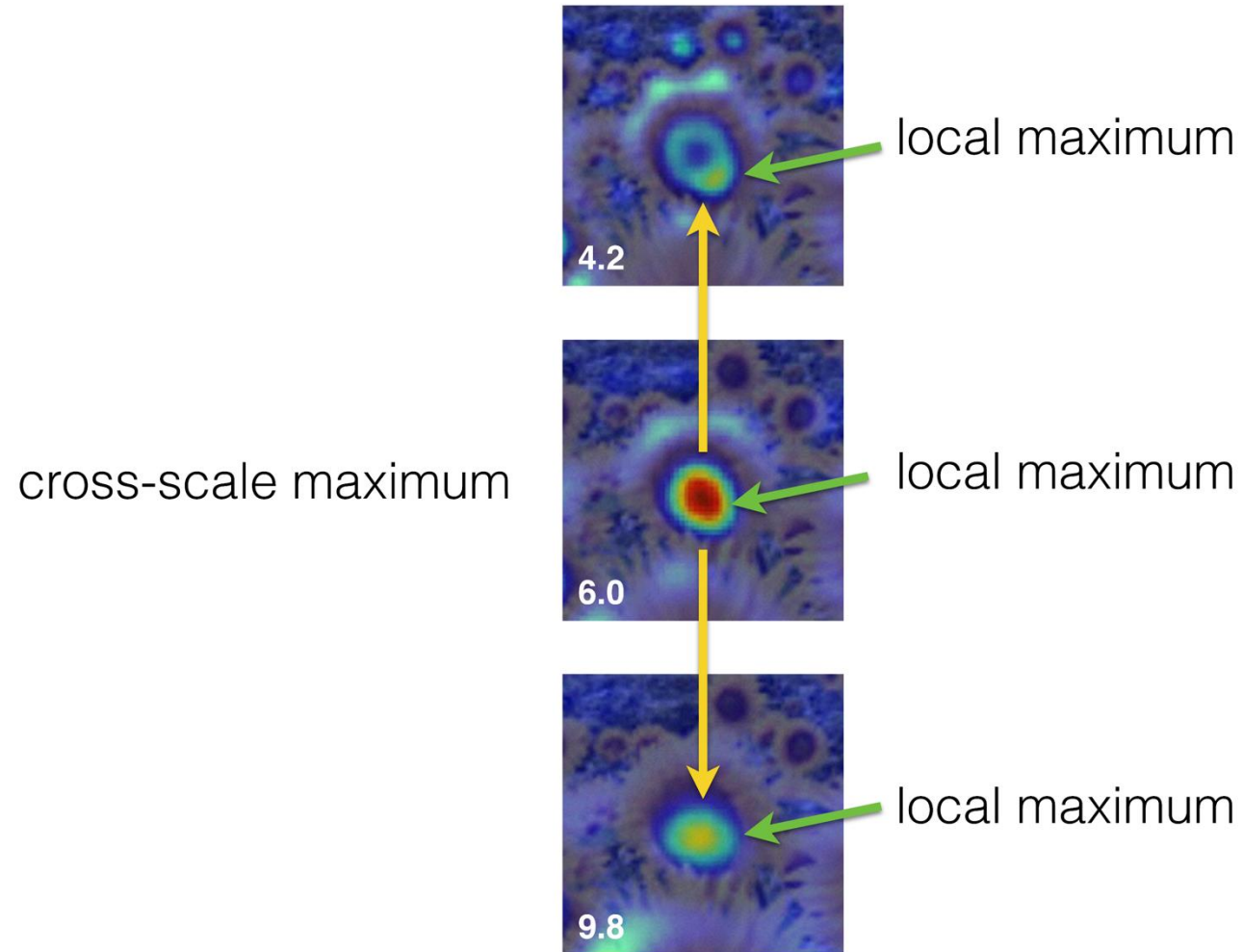
17.0



Multi-scale
2D Blob detection

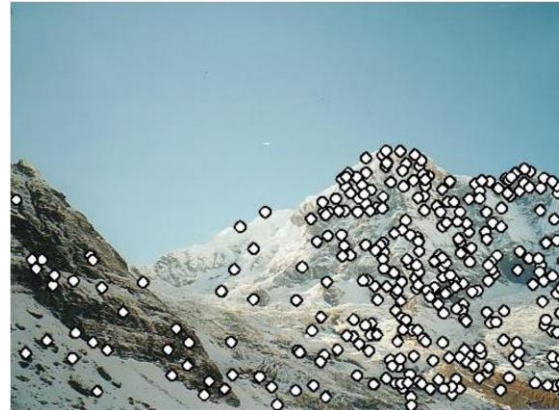


Laplacian of Gaussian for Scale Selection

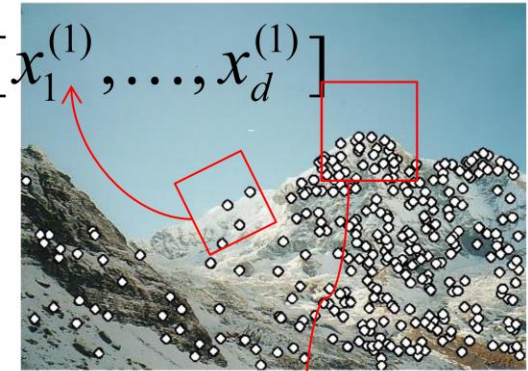


Scale Invariance Feature Transform (SIFT)

Keypoint detection



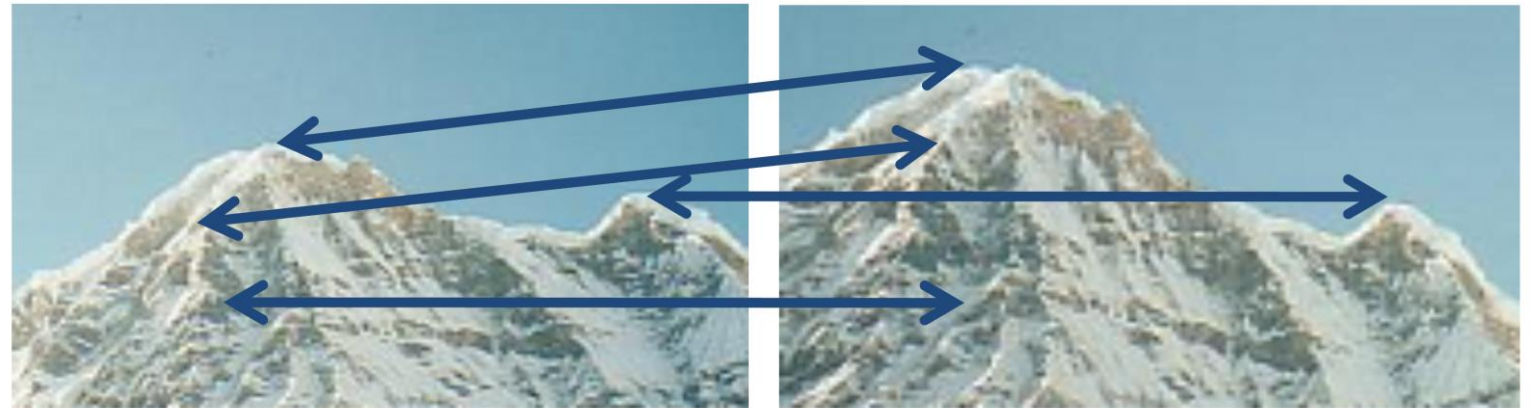
$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



Compute descriptors

$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

Matching descriptors



David Lowe, Distinctive Image Features from Scale-Invariant Keypoints. IJCV, 2004

SIFT: Scale-space Extrema Detection

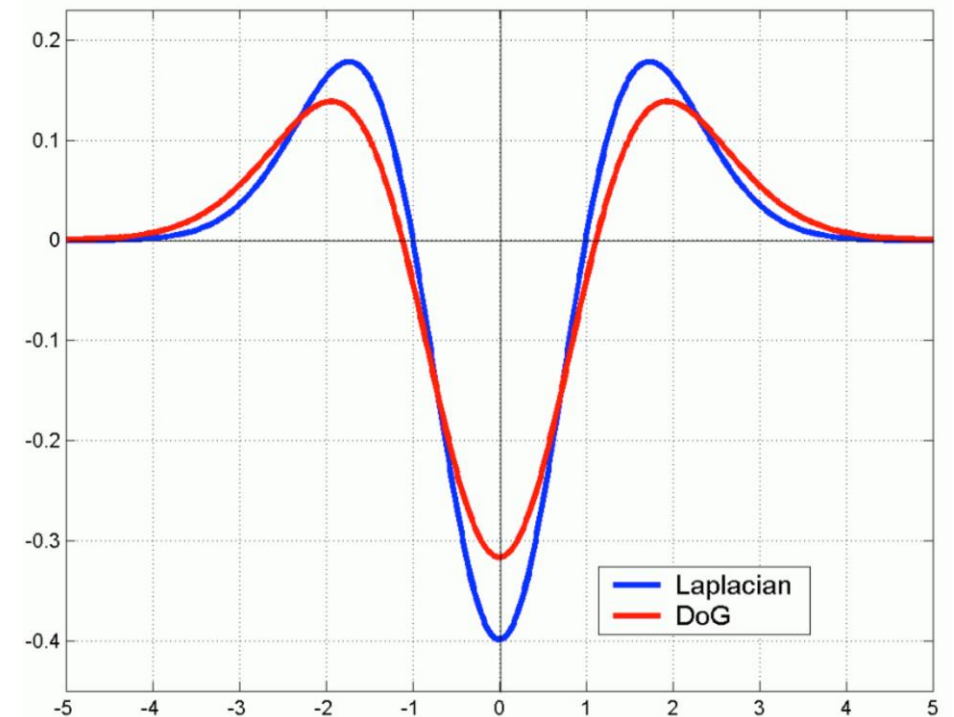
Difference of Gaussian (DoG)

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

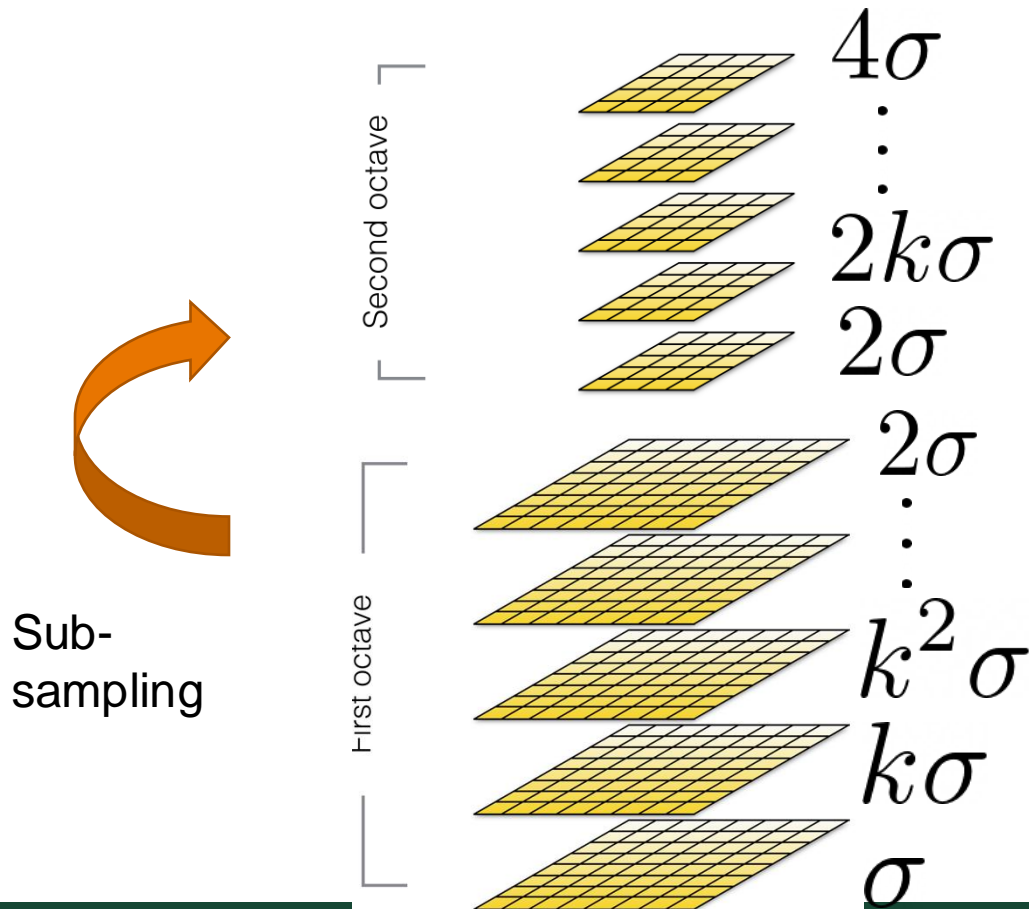
$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

Approximate of Laplacian of Gaussian
(efficient to compute)



SIFT: Scale-space Extrema Detection

Gaussian pyramid



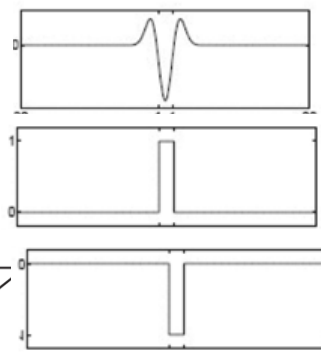
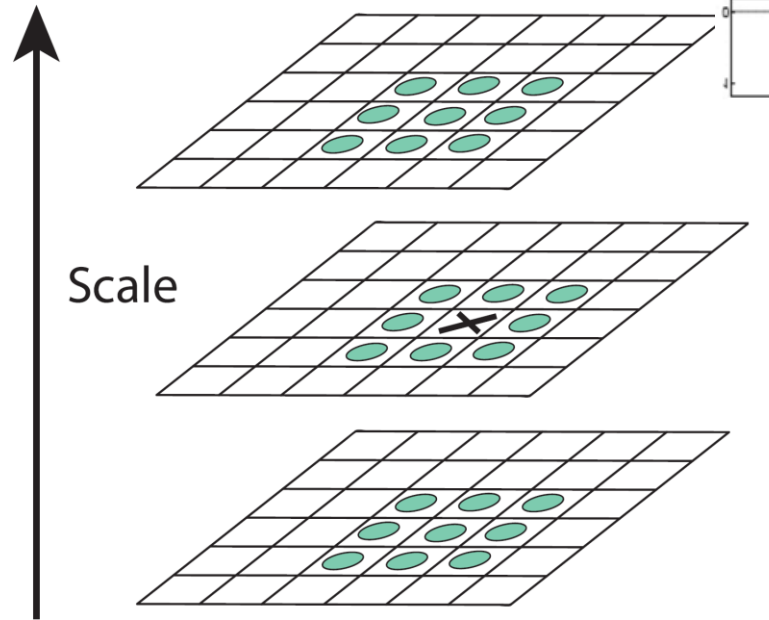
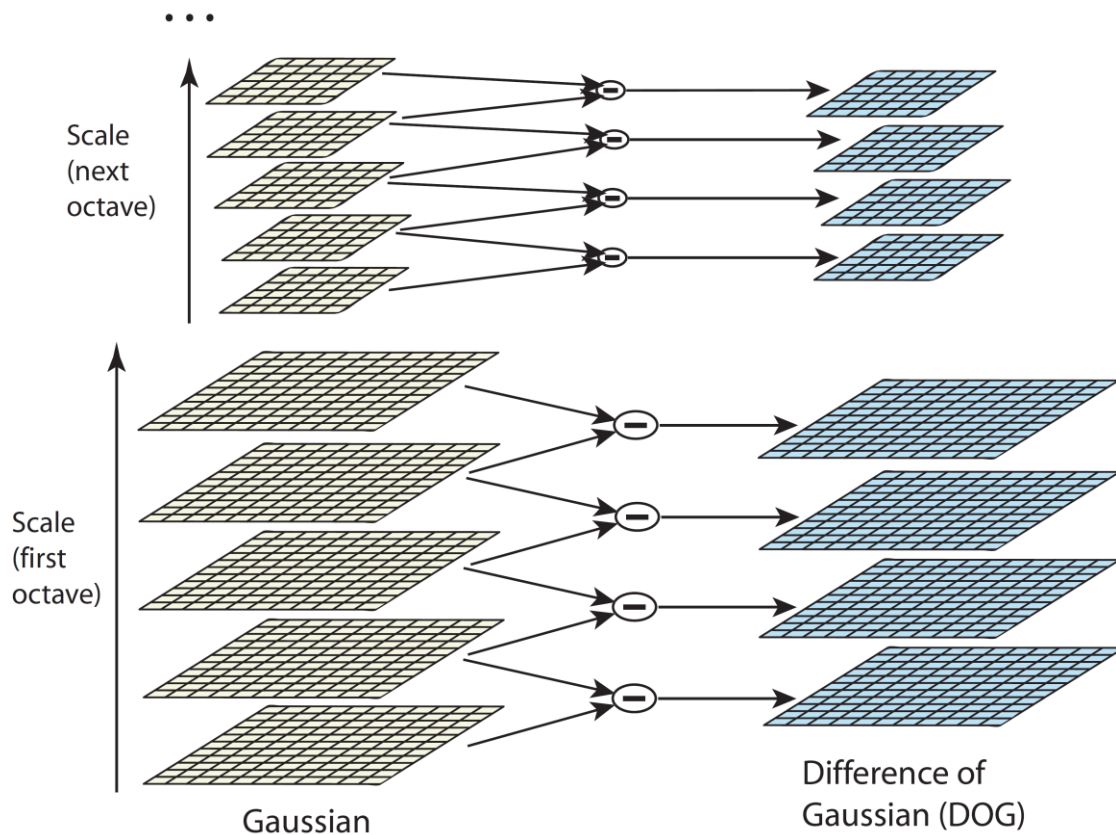
- Gaussian filters

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

$$G_{\sigma_1} * G_{\sigma_2} = G_{\sigma} \quad \sigma^2 = \sigma_1^2 + \sigma_2^2$$

- Sub-sampling by a factor of 2
 - Multiple the Gaussian kernel deviation by 2

SIFT: Scale-space Extrema Detection



Maxima and minima of DoG images

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y)$$

$$= L(x, y, k\sigma) - L(x, y, \sigma).$$

SIFT Descriptor

Image gradient magnitude and orientation

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

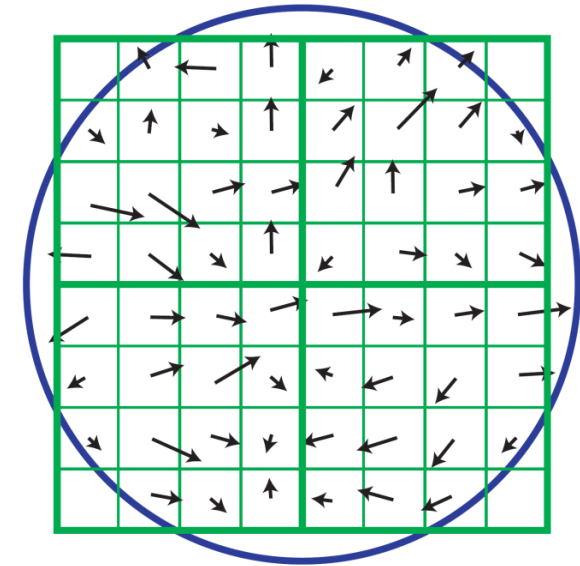


Image gradients

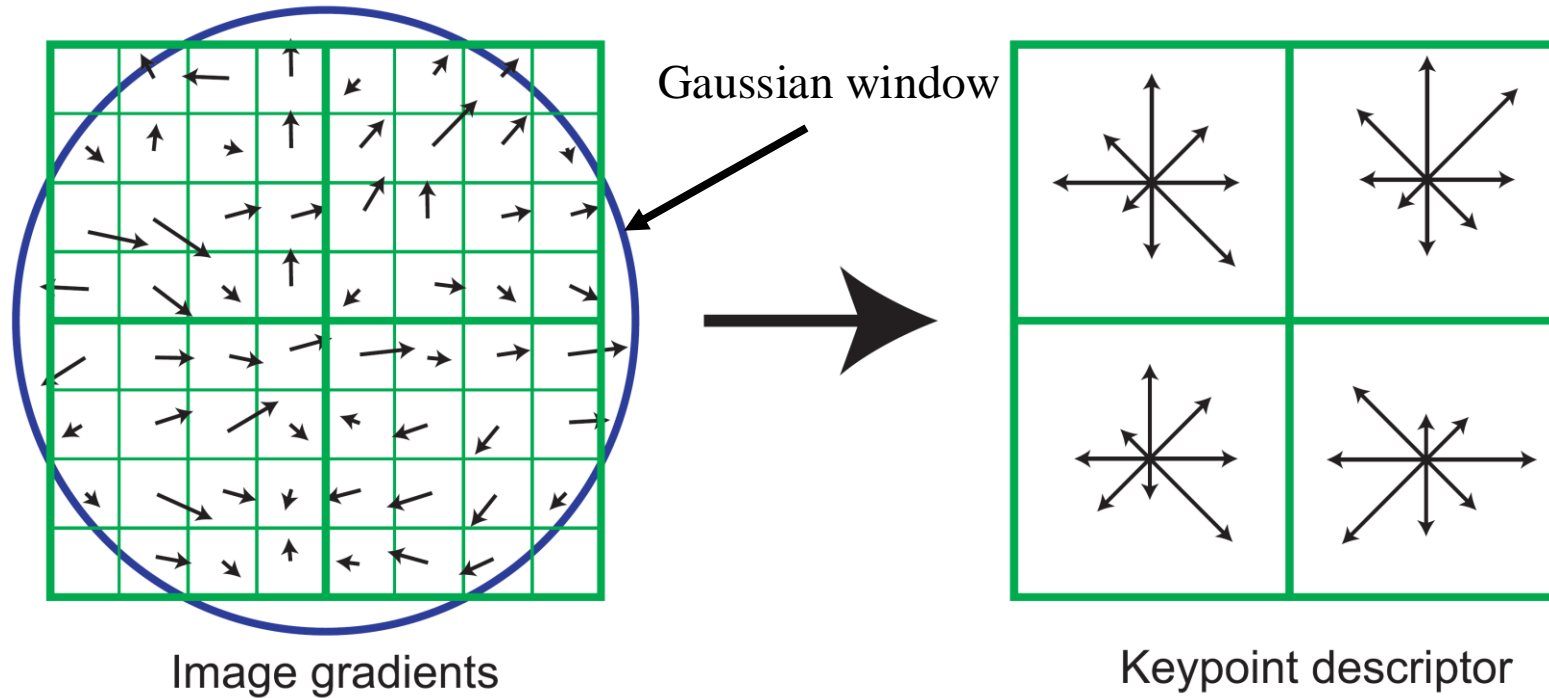
$$m(x, y) = \sqrt{\underbrace{(L(x + 1, y) - L(x - 1, y))^2}_{\text{X-derivative}} + \underbrace{(L(x, y + 1) - L(x, y - 1))^2}_{\text{Y-derivative}}}$$

$$\theta(x, y) = \tan^{-1}((L(x, y + 1) - L(x, y - 1)) / (L(x + 1, y) - L(x - 1, y)))$$

SIFT Descriptor

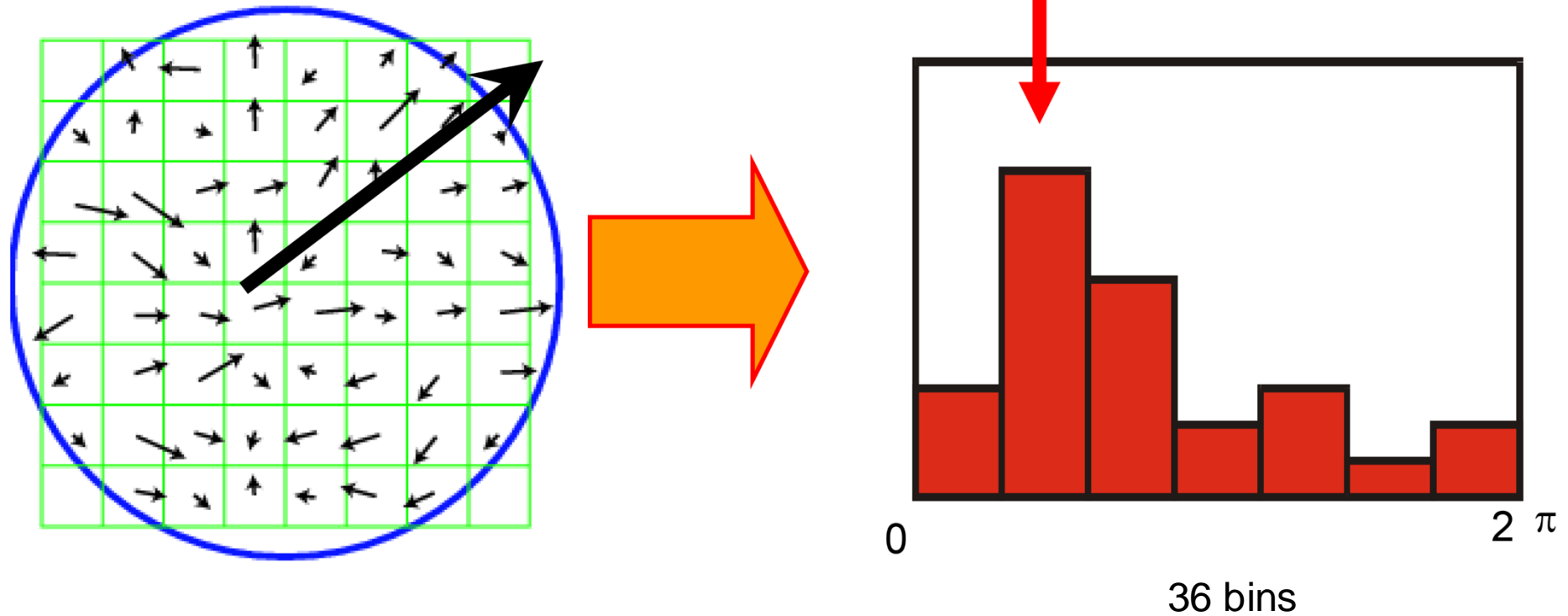
- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor

Using the scale of the keypoint to select the level of Gaussian blur for the image



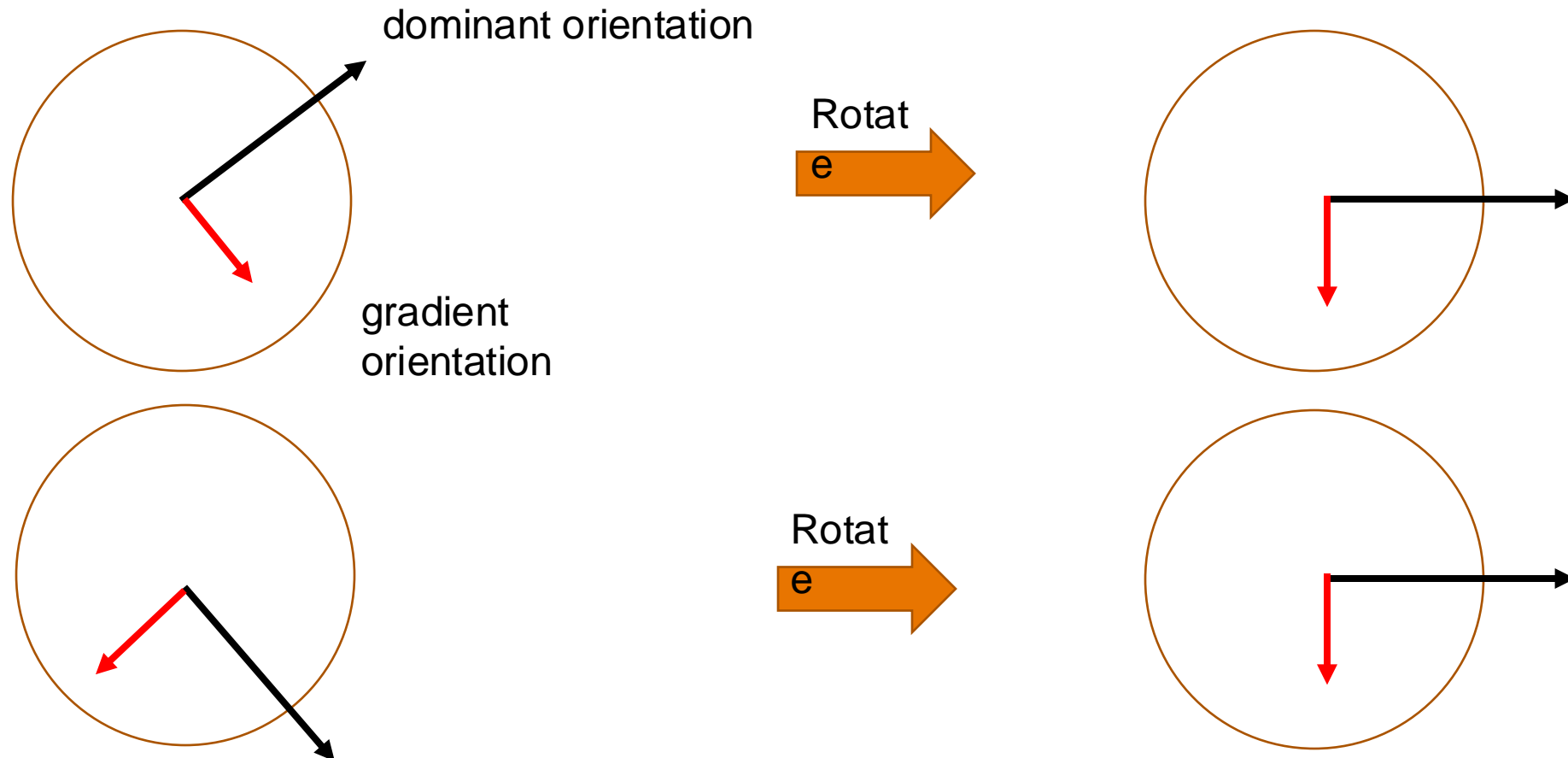
SIFT: Rotation Invariance

Rotate all orientations by the dominant orientation



SIFT: Rotation Invariance

Rotate all orientations by the dominant orientation



SIFT Properties

Can handle change in viewpoint (up to about 60 degree out of plane rotation)

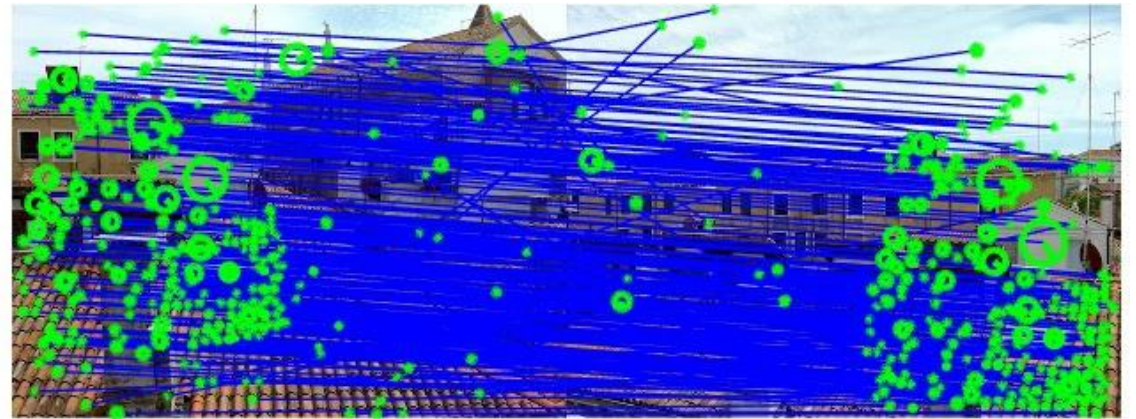
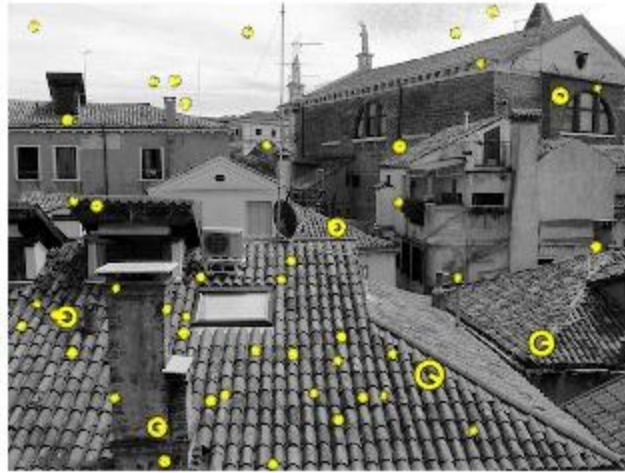
Can handle significant change in illumination

Relatively fast < 1s for moderate image sizes

Lots of code available

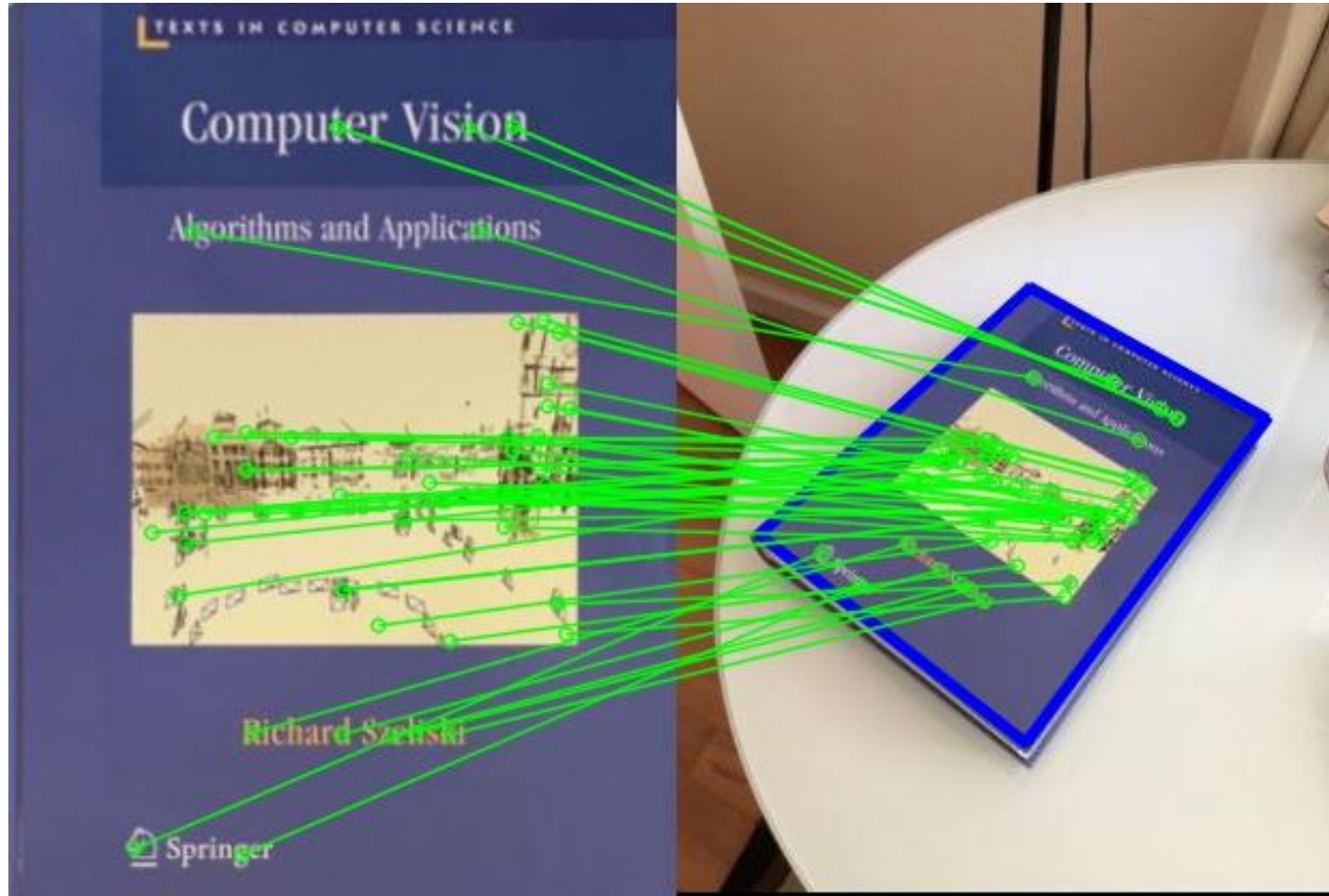
- E.g., <https://www.vlfeat.org/overview/sift.html>

SIFT Matching Example



<https://www.vlfeat.org/overview/sift.html>

SIFT Matching Example



Further Reading

Section 7.1, Computer Vision, Richard Szeliski

David Lowe, Distinctive Image Features from Scale-Invariant Keypoints. IJCV, 2004

ORB: An efficient alternative to SIFT or SURF. Rublee et al., ICCV, 2011